

This is a sample which very roughly reflects the structuring and style of questions in the exams.

You are supposed to answer ALL questions.

You can expect three or four questions. Each of them is divided into several parts.

Make sure to show your work or argumentation. The exam will be marked for the logic/calculations of your work as well as for correct results.

1. (a) Evaluate the infinite sum

$$\sum_{m=1}^{\infty} \left(-\frac{2}{3}\right)^m$$

[6 marks]

- (b) Sketch a plot of the function

$$h(x) = \frac{2|x|}{x}$$

What is the slope of the function for $x > 0$ and for $x < 0$?

Use your answers for the slope to sketch a plot of the derivative $h'(x)$.

[7 marks]

- (c) Consider the function

$$g(x) = \frac{x}{x+1}$$

What value does the function approach when x becomes large and positive?

What value does the function approach when x becomes large and negative?

Does the function diverge for any value of x ?

Using your answers to sketch a plot of the function.

[7 marks]

- (d) Two unbiased six-faced dice are thrown. Find the probability of the sum of the two results being exactly 6.

[5 marks]

2. (a) Explain how you can prove a statement for all positive integers using mathematical induction.

Prove using mathematical induction that

$$1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n+1)$$

for any positive integer n .

[15 marks]

- (b) A jar contains 5 black marbles and 5 green marbles. Two marbles are chosen blindly from the drawer. Find the probability of obtaining one marble of each colour.

[10 marks]

3. (a) Differentiate the following functions with respect to x .

(i) $x^2 - \frac{1}{x}$

(ii) $x^3 \ln(x)$

(iii) $\sin(e^x)$

[15 marks]

(b) Consider the definite integral

$$\int_{-1}^3 3|x|dx.$$

Sketch a plot of the function being integrated and identify the area represented by the integral.

By calculating the area geometrically, calculate the integral.

[10 marks]

4. (a) Consider the infinite series

$$Q = x + 2x^2 + 3x^3 + 4x^4 + \dots = \sum_{n=1}^{\infty} nx^n$$

Subtract xQ from Q . Hence derive an expression for Q .

[8 marks]

- (b) Write the following linear system of equations as a matrix equation.

$$\begin{aligned} 4x - y &= -2 \\ -x + 5y &= 3 \\ -3x - 2y &= 1 \end{aligned}$$

How many solutions does this system of equations have? Explain your answer.

[7 marks]

- (c) A 3D printer nozzle is moving horizontally along a straight line. If its position as a function of time is given by

$$x(t) = 4 - e^{-t}$$

then find the velocity and acceleration of the component, as functions of time.

What is the position of the component at very late times (large positive values of t)? Explain.

[10 marks]