

This is a sample which very roughly reflects the structuring and style of questions in the exams.

You are supposed to answer ALL questions.

You can expect three or four questions. Each of them is divided into several parts.

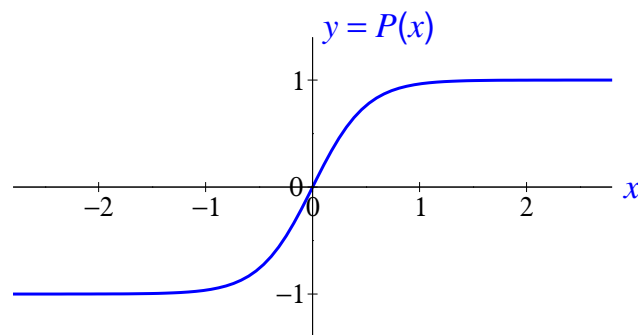
Make sure to show your work or argumentation. The exam will be marked for the logic/calculations of your work as well as for correct results.

1. (a) Evaluate the sum of the following finite arithmetic series:

$$2 + 5 + 8 + \cdots + 98$$

[7 marks]

- (b) Consider the function  $P(x)$ , plotted in the figure.



- (i) Sketch a plot of the function  $Q(x) = P(x) + 2$ , as a function of  $x$ .
- (ii) Sketch a plot of the function  $R(x) = P(x - 1)$ , as a function of  $x$ .
- (iii) Sketch a plot of the derivative  $P'(x)$ , as a function of  $x$ .
- (iv) Sketch a plot of the second derivative  $P''(x)$ , as a function of  $x$ .

[18 marks]

2. (a) Differentiate the following functions with respect to  $x$ .

(i)  $3x - \frac{1}{\sqrt{x}}$

(ii)  $x^3 \sin(x)$

(iii)  $e^{\sqrt{x}}$

[15 marks]

(b) Three unbiased coins are thrown. Find the probability of obtaining exactly two heads.

[4 marks]

(c) You are given 7 balls of different colour. In how many distinct ways could you choose 2 balls out of the set, without caring about the ordering within the two chosen balls?

[6 marks]

3. (a) Find the critical points of the function

$$f(x) = \frac{1}{3}x^3 - 9x$$

and determine whether each is a local maximum or a local minimum.

**[15 marks]**

- (b) Calculate the definite integral

$$\int_{-1}^1 x dx .$$

By sketching a plot of the function being integrated and identifying the area represented by the integral, explain your result geometrically.

**[10 marks]**

4. (a) Prove that

$$\int [xe^{-x} - e^{-x}] dx = -xe^{-x} + C$$

by remembering that integration and differentiation are inverse operations.

[7 marks]

- (b) Given the matrices

$$C = \begin{pmatrix} 0 & -2 \\ 3 & 1 \end{pmatrix} \quad \text{and} \quad D = \begin{pmatrix} \alpha \\ \beta \end{pmatrix},$$

Find out whether  $CD$  and  $DC$  are meaningful matrix products. If any of these products are meaningful, calculate it.

[7 marks]

- (c) In an automated production factory, a needle at the end of a robotic arm moves along a straight line. If its position as a function of time is given by

$$x(t) = 4t - \frac{1}{3}t^3$$

find the time instant when the needle momentarily stops.

Find the position and acceleration of the needle at this instant.

You may neglect negative times for this problem, i.e., consider only  $t \geq 0$ .

[11 marks]