Due before tutorial, monday November 19th.

Problems titled **[SELF]** are for your own practice and will not be marked.

If any calculations are required to obtain your answers, please show them. Your work will be marked for your reasoning/calculations as well as for giving the correct final answer.

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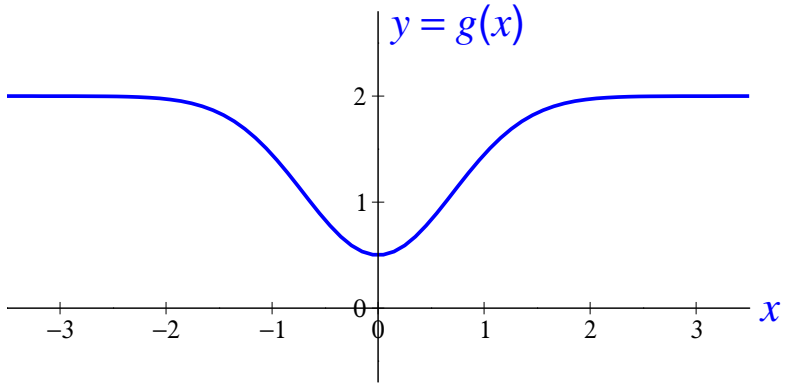
1. Consider the function $g(x)$, plotted in the figure.



   (a) **[2 pts.]** Sketch a plot of the function $f(x) = g(x) - 2$, as a function of x.

   (b) **[3 pts.]** Sketch a plot of the function $h(x) = g(x+1)$, as a function of x.

   (c) **[5 pts.]** Sketch a plot of the derivative $g'(x)$, as a function of x.

2. Consider the function

$$f(x) = \frac{1}{x^3 + 2x}\,.$$

   We will calculate the derivative in two ways.

   (a) **[3 pts.]** Defining $g(x) = 1$ and $h(x) = x^3 + 2x$, we can write

$$f(x) = \frac{g(x)}{h(x)}\,.$$

   Hence use the **quotient rule** to calculate the derivative $f'(x)$.

   (b) **[4 pts.]** If you define $u(x) = x^3 + 2x$ and $g(u) = \dfrac{1}{u}$, then

$$f(x) \;=\; g(u(x))\,.$$

   Use the **chain rule** to calculate the derivative $f'(x)$.

3. (a) **[SELF]** Find the critical point(s) of the function $f(x) = 3+4x-2x^2$. How many critical points does this function have?

   (b) **[SELF]** For each critical point, find out using the second derivative whether the function is a minimum or a maximum.

   (c) **[SELF]** In which region (for which values of x) is the curve $f(x)$ concave downwards, and for which region is it convex downwards?

   (d) **[SELF]** Express the function in the form $a+b(x-c)^2$. What are the values of a, b, and c? Use this form to plot the graph of the function.

4. (a) **[4 pts.]** Find the critical point(s) of the function

$$f(x) = x^3 - 6x + 1\,.$$

   How many critical points does this function have?

   (b) **[4 pts.]** For each critical point, find out using the second derivative whether the function is a minimum or a maximum.

   (c) **[2 pts.]** In which region (for which values of x) is the curve $y = f(x)$ concave downwards, and for which region is it convex downwards?

5. (a) **[SELF]** Sketch a plot of the function $f(x) = e^{-x}$ without using a calculator. Hint: think about the behavior of the function at $x = 0$, at large positive x, and at large negative x.

   Use this knowledge to plot the function $g(x) = x\,e^{-x}$, without using a calculator. Hint: think about the behavior of the function at $x = 0$, at large positive x, and at large negative x.

   (b) **[4 pts.]** Find the critical point(s) of the function

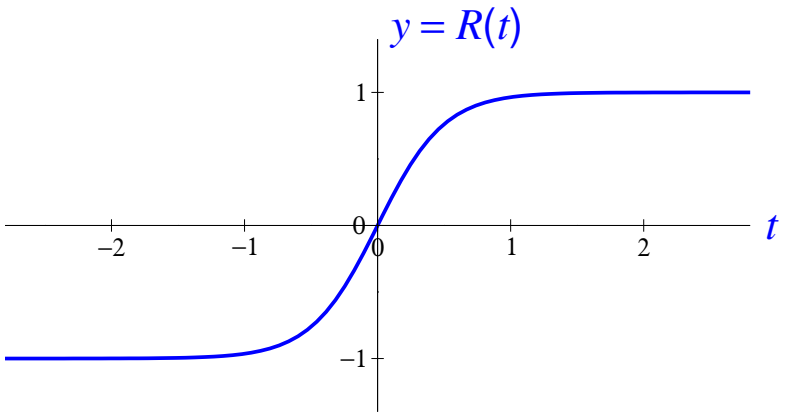
$$g(x) = x\ e^{-x}\,.$$

   How many critical points does this function have?

   (c) **[4 pts.]** For each critical point, find out using the second derivative whether the function is a minimum or a maximum.

   (d) **[2 pts.]** In which region (for which values of x) is the curve $y = f(x)$ concave downwards, and for which region is it convex downwards?

6. Consider the function $R(t)$, plotted in the figure.



(a) **[4 pts.]** Sketch a plot of the function $S(t) = R(t-2)+1$, as a function of t.

(b) **[4 pts.]** Sketch a plot of the derivative, $R'(t)$, as a function of t.

(c) **[5 pts.]** Sketch a plot of the second derivative or double derivative, $R''(t)$, as a function of t.