Due before tutorial, monday November 12th.

Problems titled [**SELF**] are for your own practice and will not be marked.

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You might need to know:

$$\frac{d}{dx}\sin(x) \;=\; \cos(x)\,; \qquad \frac{d}{dx}\cos(x) \;=\; -\sin(x)\,; \qquad \frac{d}{dx}\ln(x) \;=\; \frac{1}{x}$$

Here $\ln(x)$ or $\ln x$ means the natural logarithm of x, i.e., the logarithm of x to the base e, as discussed in lecture.

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1. Consider the function

$$g(x) = 3x^2 - 2x$$

   (a) [**2 pts.**] Use the rules of differentiation to calculate the derivative $g'(x)$.

   (b) [**7 pts.**] Calculate $g'(x)$ from first principles, and show that you get the same result.
   Reminder: calculating derivatives by first principles means using the definition $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$.

2. Consider the function

$$h(x) = \frac{x + 2\sqrt{x}}{x^3}\,.$$

   This function is defined only for $x > 0$.

   (a) [**5 pts.**] Defining $f(x) = x + 2\sqrt{x}$ and $g(x) = x^3$, we can write

$$h(x) = \frac{f(x)}{g(x)}\,.$$

   Hence use the **quotient rule** to calculate the derivative $h'(x)$.

   (b) [**3 pts.**] We can simplify to get

$$h(x) \;=\; \frac{1}{x^2} + 2x^{-5/2}\,.$$

   From this expression, calculate the derivative $h'(x)$. You don't need to use the quotient rule.

3. Find the derivatives of the following functions using the rules for derivatives. Please remember to show your calculations. You don't need the chain rule.

(a) **[3 pts.]** $f(x) = 2x^3 \ln(x)$

(b) **[3 pts.]** $f(x) = e^x \cos(x)$

(c) **[4 pts.]** $f(x) = 2x^3 \sin(x) + xe^x$

(d) **[4 pts.]** $f(x) = \dfrac{\cos(x)}{2x^3}$

(e) **[SELF]** $f(x) = 2xe^x$

(f) **[SELF]** $f(x) = (x^2 - 2x)\sin(x)$

(g) **[SELF]** $f(x) = (2x^3 - 3)\ln(x)$

(h) **[SELF]** $f(x) = (-3x^2 - x)e^x$

(i) **[SELF]** $f(x) = 2x\sin(x)$

4. Find the derivatives of the following functions using the rules of differentiation. Please remember to show your calculations. Now you will also need the chain rule.

(a) **[4 pts.]** $f(x) = \sqrt{x^3 + \dfrac{5}{x}}$

(b) **[4 pts.]** $f(x) = \cos(2x^3)$

(c) **[5 pts.]** $f(x) = \dfrac{2}{x^2} e^{-5x^2}$

(d) **[6 pts.]** $f(x) = \sin\left(e^{x^2}\right)$

(e) **[SELF]** $f(x) = e^{-5x^2}$

(f) **[SELF]** $f(x) = \dfrac{3}{4}x^4 - \ln(\sin x)$

(g) **[SELF]** $f(x) = (\ln(x^2 - 3))(e^{2x})$

(h) **[SELF]** $f(x) = \exp\left[\dfrac{x+2}{x^2 - 3}\right]$ $\left\{\begin{array}{l}\text{Note: } \exp[u] \text{ is} \\ \text{another way to write } e^u.\end{array}\right.$

(i) **[SELF]** $f(x) = \cos\left(x^2 + \sqrt{2x - \ln x}\right)$

(j) **[SELF]** $f(x) = \cos\left(x^2 - \dfrac{3}{x}\right)$