Due at the beginning of the first tutorial, monday 1st October, 10AM.

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1. The sequence $\{a_n\}_{n=1}^{\infty}$ is defined by the recurrence relation

$$a_1 = 2 \qquad \text{and} \qquad a_n = (a_{n-1})^2 + 1 \quad \text{for} \quad n > 1$$

   (a) [**5 pts.**] Write down the first five terms of the sequence $\{a_n\}$.

   (b) [**5 pts.**] Another sequence $\{b_n\}_{n=1}^{\infty}$ is defined in terms of the sequence above:

$$b_n = a_{n+1} - a_n$$

   Write down the first four terms of this new sequence.

2. Consider the sequence $\{F_n\}_{n=1}^{\infty}$ defined by

$$F_1 = F_2 = 1 \qquad \text{and} \qquad F_n = F_{n-2} + F_{n-1} \quad \text{for} \quad n > 2$$

   (a) [**4 pts.**] This is known as the Fibonacci sequence. Write down the first 10 terms.

   (b) [**6 pts.**] It is claimed that the Fibonacci sequence can also be described as

$$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$

   Check this claim for $n = 1$, $n = 2$, and $n = 3$, by evaluating this formula for these values and comparing with the sequence members you wrote down previously.

3. [**5+5 pts.**] Find the formula for the n-th term of each arithmetic sequence below.

   (a) 2, 5, 8, ...

   (b) 107, 98, 89, ...

4. (a) [**6 pts.**] Provide a plot of the sequence $\{g_n\}$ defined by

$$g_n = \frac{1}{n}$$

You can choose to produce a plot using a computer program, or to provide a neat drawing produced by hand.

Of course, you cannot show n up to infinity on a finite sheet of paper, so I suggest you show up to $n = 10$.

(b) [**4 pts.**] What value does the sequence $\{g_n\}$ converge to?

5. (a) [**6 pts.**] Provide a plot of the finite sequence $\{a_n\}_{n=1}^{10}$ defined by

$$a_n = (-1)^n \, n$$

(b) [**4 pts.**] Consider the infinite sequence $\{a_n\}_{n=1}^{\infty}$ defined by

$$a_n = (-1)^n \, n$$

Does this sequence converge?