MP465 – Advanced Electromagnetism

Lecture 17 & 18 Part II (9 April 2020)

2. General Consequences of the Boundary Conditions

What we'll now do is look at the consequences of these for a very realistic case, the case where we have a known EM plane wave travelling through a medium and encountering another medium. More specifically, consider the case mentioned a while ago where the region $z < 0$ is filled with medium 1 and $z > 0$ filled with medium 2. We now send a known EM plane wave from inside medium 1 and try to figure out what the fields everywhere in space are.

So we assume we have an incident monochromatic EM plane wave which we are free to pick as we choose. (Think of setting up a laser and blasting some material with its beam.) Thus, we know its frequency ω, its wave vector $\vec{k}_I$ and its polarisation given by some complex electric amplitude vector $\tilde{\vec{E}}_0$ so that the fields are

$$\vec{E}_I(t,\vec{r}) = \mathrm{Re}\left[\tilde{\vec{E}}_0 e^{i(\vec{k}_I\cdot\vec{r}-\omega t)}\right], \qquad \vec{B}_I(t,\vec{r}) = \frac{\vec{k}_I}{\omega}\times\vec{E}_i(t,\vec{r}).$$

Recall we've picked a coordinate system where the boundary is the xy-plane, but this only specifies the z-axis, not the x- or y axes. Thus, without loss of generality, we can pick these axes so that the incident wave vector $\vec{k}_I$ lies in the xz-plane, giving us the diagram below.

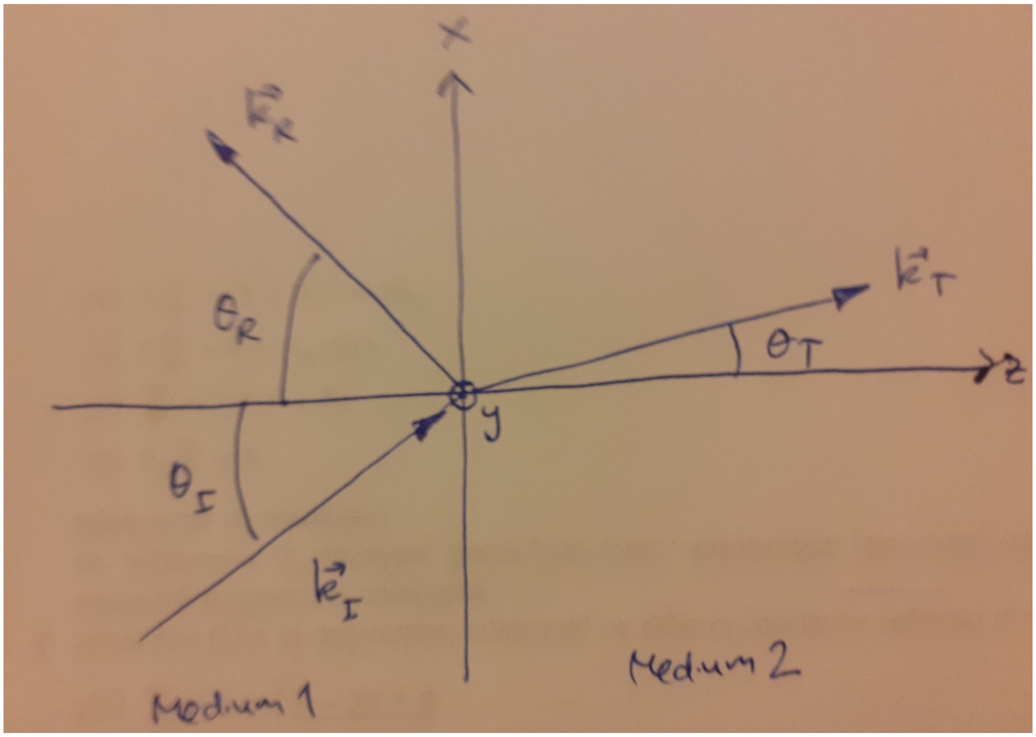
Note that this now allows us to define the angle of incidence θ_I as the angle between $\vec{k}_I$ and the z-axis, meaning that we can write

$$\vec{k}_I \;\; = \;\; \frac{n_1\omega}{c}\left(\sin\theta_I\,\hat{e}_x + \cos\theta_I\,\hat{e}_z\right)$$

because this wave is in medium 1 and therefore $|\vec{k}_I| = \omega/v_1 = n_1\omega/c$. If we put this wave vector in explicitly, then that gives

$$\vec{E}_I(t,\vec{r}) \;\; = \;\; \mathrm{Re}\left\{\tilde{\vec{E}}_0 \exp\left[\frac{i\omega}{c}\left(n_1\sin\theta_I\,x + n_1\cos\theta_I\,z - ct\right)\right]\right\}.$$

Now, let's invoke some results we saw in MP205: if a wave travels along a string with a given mass density, but then encounters a part of the string with



a different mass density, then is splits into a backwards-travelling reflected wave and a forward-travelling transmitted wave. We assume the same happens here, namely, that as a result of the incident wave hitting the boundary, there is a reflected wave in medium 1 of the form

$$\vec{E}_R(t,\vec{r}) = \mathrm{Re}\left[\tilde{\vec{E}}_{0R} e^{i(\vec{k}_R\cdot\vec{r}-\omega_R t)}\right], \qquad \vec{B}_R(t,\vec{r}) = \frac{\vec{k}_R}{\omega}\times\vec{E}_R(t,\vec{r}).$$

and a transmitted wave in medium 2 of the form

$$\vec{E}_T(t,\vec{r}) = \mathrm{Re}\left[\tilde{\vec{E}}_{0T} e^{i(\vec{k}_T\cdot\vec{r}-\omega_T t)}\right], \qquad \vec{B}_T(t,\vec{r}) = \frac{\vec{k}_T}{\omega}\times\vec{E}_T(t,\vec{r}).$$

In principle, the two wave vectors can point in any directions, so in general we can say is that that phases in the exponentials may be written as

$$\begin{aligned}\vec{k}_R\vec{r} - \omega_R t &= k_{Rx}x + k_{Ry}y + k_{Rz}z - \omega_R t,\\ \vec{k}_T\vec{r} - \omega_T t &= k_{Tx}x + k_{Ty}y + k_{Tz}z - \omega_R t\end{aligned}$$

with the only obvious restrictions being that $k_{Rz} < 0$ (so that it's travelling from the boundary through medium 1) and $k_{Tz} > 0$ (so that it's travelling from the boundary through medium 2).

But we just talked about the boundary conditions all the fields have to satisfy. In this case, they're all at $z = 0$, but much hold over the *entire* boundary and at *all* times, i.e. for all values of x, y and t. But at the boundary, the phase of the incident wave is $n_1\omega \sin\theta_i \, x/c - \omega t$, which means that the phases of all other waves must have the same form: specifically, they can't have any y-dependence and their time dependence must be $-\omega t$. Thus, none of the wave vectors can have a y-component and thus are all coplanar (in this case all in the xz-plane). Plus, all frequencies are the same; the reflected and transmitted waves are the same colour as the incident wave.

If they're all coplanar, this means we can define the angle of reflection θ_R and angle of transmission θ_R as shown and thus $|\vec{k}_R| = \omega/v_1 = n_1\omega/c$ and and $|\vec{k}_T| = \omega/v_2 = n_2\omega/c$. All of this gives explicit expression for the two new wave vectors:

$$\begin{aligned}
\vec{k}_R &= \frac{n_1\omega}{c}\left(\sin\theta_R \, \hat{e}_x - \cos\theta_R \, \hat{e}_z\right), \\
\vec{k}_T &= \frac{n_2\omega}{c}\left(\sin\theta_T \, \hat{e}_x + \cos\theta_T \, \hat{e}_z\right)
\end{aligned}$$

But by again invoking the matching of the phases at the boundary, we note that the x-dependence of all three waves must also be the same at $z = 0$, and this gives us possibly the two most famous theorems in classical optics! First, if the x-dependence of the incident and reflected wave must be the same, then $n_1\omega \sin\theta_R \, x/c = n_1\omega \sin\theta_I \, x/c$, or $\theta_I = \theta_R$: the angle of incidence is equal to the angle of reflection. A light beam bounces off a surface at the same angle as it hits it.

Second, the matching of the x-dependence of the incident and transmitted wave implies $n_2\omega \sin\theta_T \, x/c = n_1\omega \sin\theta_I \, x/c$, or $n_1 \sin\theta_I = n_2 \sin\theta_T$. This relation between the transmission angle and incidence angle is known as *Snell's law* and has been known empirically for centuries. It implies that if $n_1 \neq n_2$ then $\theta_T \neq \theta_I$ and the light beam changes direction when it moves across the boundary; it *refracts*. This is precisely why we use the term "index of refraction" (and also explains the cover image of *Dark Side of the Moon*).

So from this very general argument, we have derived the following basic facts about what happens when a EM wave hits a boundary, and we can take them as given (i.e. we don't have to rederive them every single time):

all waves have the same frequency, all the wave vectors are coplanar and, with the angles defined as above, $\theta_I = \theta_R$ and $n_1 \sin\theta_I = n_2 \sin\theta_T$.

The main reason for initially taking care of the spacetime dependence first is that now our boundary conditions need only be imposed on the amplitude vectors of the various fields, namely, all these constant complex vectors flying around the place.

To see how this works we see that in medium 1, the total electric field is $\vec{E}_I + \vec{E}_R$ and in medium 2 it's $\vec{E}_T$, so at the boundary $z = 0$,

$$
\begin{aligned}
\vec{E}_1 &= \left.\vec{E}_I + \vec{E}_R\right|_{z=0} \\
&= \operatorname{Re}\left\{\left(\tilde{\vec{E}}_0 + \tilde{\vec{E}}_{0R}\right)\exp\left[\frac{i\omega}{c}\left(n_1 \sin\theta_I\, x - \omega t\right)\right]\right\}, \\
\vec{E}_2 &= \left.\vec{E}_T\right|_{z=0} \\
&= \operatorname{Re}\left\{\tilde{\vec{E}}_{0T}\exp\left[\frac{i\omega}{c}\left(n_1 \sin\theta_I\, x - \omega t\right)\right]\right\},
\end{aligned}
$$

where we've used $\theta_I = \theta_R$ and Snell's law. Thus, since the spacetime dependence is the same for both fields at the boundary, any conditions they must satisfy must be satisfied purely by the constant amplitude vectors. The same argument holds when we look at the magnetic, electric displacement and magnetic intensity fields: the sum of the incident and reflected amplitude vectors and the transmitted amplitude vector must satisfy the appropriate boundary condition.

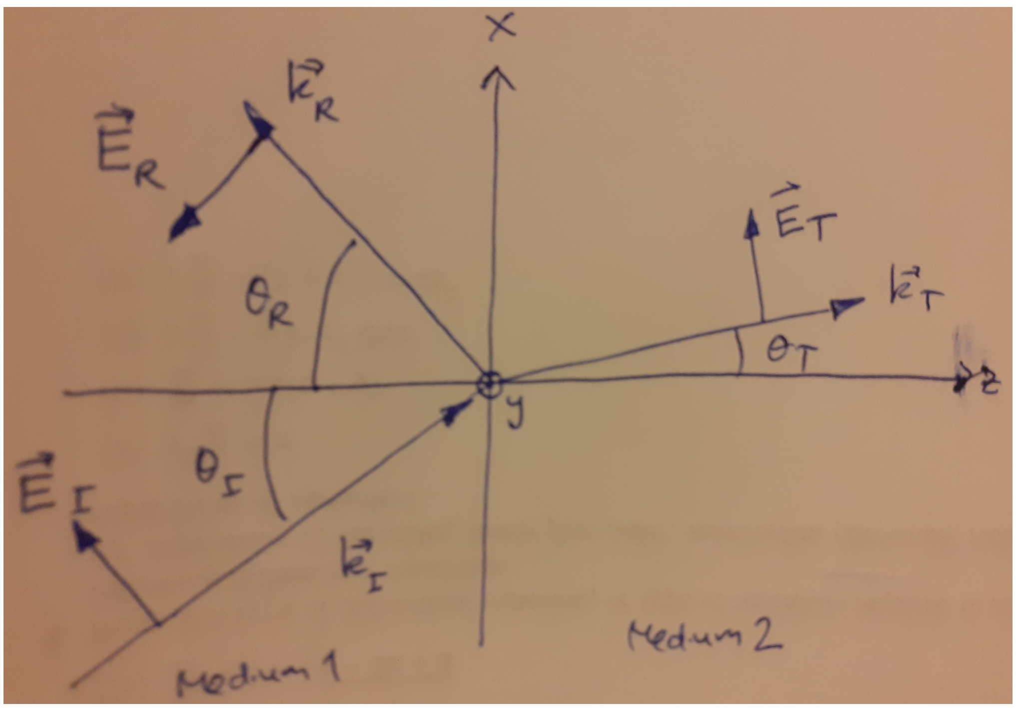
And this will completely solve the problem: we're given the properties of each medium (permittivities, etc.), an initial frequency ω, and initial direction θ_I and an initial amplitude/polarisation $\tilde{\vec{E}}_0$. θ_R and θ_T are determined from θ_I and the two indices of refraction, leaving only $\tilde{\vec{E}}_{0R}$ and $\tilde{\vec{E}}_{0T}$ as unknowns. These are six complex numbers, but notice that that's *exactly* how many equations we have from the boundary conditions: two from the normal components of $\vec{D}$ and $\vec{B}$ and four from the parallel components of $\vec{E}$ and $\vec{H}$! We have the same number of equations as unknowns, so we're guaranteed to be able to find a unique solution.

(Well, almost; if you look at a weird situation like when the incident wave is parallel to the surface, i.e. $\theta_I = \pi/2$, you don't get a unique solution. But for all nonpathological cases, we can find a solution.)

3. An Example

The previous section describes what we'd do the general case, but instead, in this section we'll look at a specific polarisation for the incident wave and see how we use the above-described boundary conditions to solve for the fields. (Another specific – and slightly easier – choice of polarisation will be left for you to do in the next problem set.)

So what we'll do is assume that our incident electric field is linearly polarised in the xz-plane, i.e. $\tilde{\vec{E}}_0$ has no y-component. However, we know it can't be any such vector; it must be perpendicular to the wave vector $\vec{k}_I$, as shown on the next page.



It's a matter of simple trigonometry to show that this choice gives

$$\tilde{\vec{E}}_0 \quad = \quad \mathcal{E}_I \, (\cos\theta_I \, \hat{e}_x - \sin\theta_I \, \hat{e}_z)$$

where $\mathcal{E}_I$ is the incident amplitude (the strength of our laser beam, if you like). Now, there's also the reflected wave, and its amplitude vector $\tilde{\vec{E}}_{0R}$ must be perpendicular to its wave vector $\vec{k}_R$. This condition only gives a relation between its x- and z-components, since any component in the y-direction will automatically be normal to $\vec{k}_R$. Thus, the most general form is

$$\tilde{\vec{E}}_{0R} = -\mathcal{E}_R\left(\cos\theta_R\,\hat{e}_x + \sin\theta_R\,\hat{e}_z\right) + \tilde{E}_{Ry}\hat{e}_y$$

for some numbers $\mathcal{E}_R$ and $\tilde{E}_{Ry}$. Similar reasoning says the the most general vector $\tilde{\vec{E}}_{0T}$ that's perpendicular to $\vec{k}_T$ is

$$\tilde{\vec{E}}_{0T} = \mathcal{E}_R\left(\cos\theta_T\,\hat{e}_x - \sin\theta_T\,\hat{e}_z\right) + \tilde{E}_{Ty}\hat{e}_y$$

Now let's impose the boundary conditions on the electric field, namely, that the two components parallel to the boundary must match across the boundary. In this case, those are the x- and y components, so we see that

$$\begin{aligned}\left(\tilde{\vec{E}}_0 + \tilde{\vec{E}}_{0R}\right)_x = \left(\tilde{\vec{E}}_{0T}\right)_x \quad &\Rightarrow \quad \mathcal{E}_I\cos\theta_I - \mathcal{E}_R\cos\theta_R = \mathcal{E}_T\cos\theta_T,\\ \left(\tilde{\vec{E}}_0 + \tilde{\vec{E}}_{0R}\right)_y = \left(\tilde{\vec{E}}_{0T}\right)_y \quad &\Rightarrow \quad \tilde{E}_{Ry} = \tilde{E}_{Ty}.\end{aligned}$$

Now for the displacement field: in a linear medium with permittivity ϵ, we know that $\vec{D} = \epsilon\vec{E}$, so in medium 1, this is $\epsilon_1(\vec{E}_I + \vec{E}_R)$ and in medium 2 it's $\epsilon_2\vec{E}_T$. The normal component of both must be identical at the boundary, and here it's the z-component. This condition must be satisfied by the amplitude vectors, giving

$$\epsilon_1\left(\tilde{\vec{E}}_0 + \tilde{\vec{E}}_{0R}\right)_z = \epsilon_2\left(\tilde{\vec{E}}_{0T}\right)_z \quad \Rightarrow \quad -\epsilon_1\left(\mathcal{E}_I\sin\theta_I + \mathcal{E}_R\sin\theta_R\right) = -\epsilon_2\mathcal{E}_T\sin\theta_T.$$

But notice that we already have two equations involving $\mathcal{E}_R$ and $\mathcal{E}_T$, so we can solve them immediately: using $\theta_R = \theta_I$ and rearranging the first and third equations above gives

$$\begin{aligned}\mathcal{E}_R + \frac{\cos\theta_T}{\cos\theta_I}\,\mathcal{E}_T &= \mathcal{E}_I,\\ \mathcal{E}_R - \frac{\epsilon_2\sin\theta_T}{\epsilon_1\sin\theta_I}\,\mathcal{E}_T = \mathcal{E}_R - \frac{n_1\epsilon_2}{n_2\epsilon_1}\,\mathcal{E}_T &= -\mathcal{E}_I.\end{aligned}$$

where we used $\sin\theta_T/\sin\theta_I = n_1/n_2$ in the the second equation. Now you can solve these simultaneous equations using whatever your favourite method is, with the result being

$$\begin{aligned}\mathcal{E}_R &= \left(\frac{n_1\epsilon_2\cos\theta_I - n_2\epsilon_1\cos\theta_T}{n_1\epsilon_2\cos\theta_I + n_2\epsilon_1\cos\theta_T}\right)\mathcal{E}_I,\\ \mathcal{E}_T &= \left(\frac{2n_2\epsilon_1\cos\theta_I}{n_1\epsilon_2\cos\theta_I + n_2\epsilon_1\cos\theta_T}\right)\mathcal{E}_I.\end{aligned}$$

(We could replace θ_T by $\arcsin(n_1 \sin\theta_I/n_2)$ if we wanted everything in terms of the incident wave parameters, but let's not and just assume we'll compute θ_T when we need it.)

Sadly, we're not done; we still need to impose the conditions on the magnetic and magnetic intensity field. These will be three more equations, but it seems like we now only have *two* unknown parameters, $\tilde{E}_{Ry}$ and $\tilde{E}_{Ty}$. We've determined $\mathcal{E}_R$ and $\mathcal{E}_T$. Luckily, two of the upcoming equations will give us results we already know, as we now see...

We now need to compute the magnetic fields for each wave, but this is easy (if a little tedious):

$$\begin{aligned}
\tilde{\vec{B}}_0 = \frac{\vec{k}_I}{\omega} \times \tilde{\vec{E}}_0 &= \frac{n_1 \mathcal{E}_I}{c} \hat{e}_y, \\
\tilde{\vec{B}}_{0R} = \frac{\vec{k}_R}{\omega} \times \tilde{\vec{E}}_{0R} &= \frac{n_1}{c} \left[\tilde{E}_{Ry} \left(\cos\theta_R \hat{e}_x + \sin\theta_R \hat{e}_z \right) + \mathcal{E}_R \hat{e}_y \right], \\
\tilde{\vec{B}}_{0T} = \frac{\vec{k}_T}{\omega} \times \tilde{\vec{E}}_{0T} &= \frac{n_2}{c} \left[\tilde{E}_{Ty} \left(-\cos\theta_T \hat{e}_x + \sin\theta_T \hat{e}_z \right) + \mathcal{E}_T \hat{e}_y \right].
\end{aligned}$$

One of our conditions is continuity of $B_\perp$ across the boundary, and here that means

$$\left(\tilde{\vec{B}}_0 + \tilde{\vec{B}}_{0R} \right)_z = \left(\tilde{\vec{B}}_{0T} \right)_z \quad \Rightarrow \quad n_1 \tilde{E}_{0R} \sin\theta_R = n_2 \tilde{E}_{0T} \sin\theta_T,$$

but if we use Snell's law, this says $\tilde{E}_{0R} = \tilde{E}_{0T}$, which we already knew. Now, since $\vec{H} = \vec{B}/\mu$, the magnetic intensity in medium 1 is $(\vec{B}_I + \vec{B}_R)/\mu_1$ and in medium 2 it's $\vec{B}_T/\mu_2$, so matching the parallel components at the boundary gives

$$\begin{aligned}
\frac{1}{\mu_1} \left(\tilde{\vec{B}}_0 + \tilde{\vec{B}}_{0R} \right)_x = \frac{1}{\mu_2} \left(\tilde{\vec{B}}_{0T} \right)_x \quad &\Rightarrow \quad \frac{n_1 \tilde{E}_{0R} \cos\theta_R}{\mu_1} = -\frac{n_2 \tilde{E}_{0T} \cos\theta_T}{\mu_2}, \\
\frac{1}{\mu_1} \left(\tilde{\vec{B}}_0 + \tilde{\vec{B}}_{0R} \right)_y = \frac{1}{\mu_2} \left(\tilde{\vec{E}}_{0T} \right)_y \quad &\Rightarrow \quad \frac{n_1}{\mu_1} \left(\mathcal{E}_I + \mathcal{E}_R \right) = \frac{n_2}{\mu_2} \mathcal{E}_T.
\end{aligned}$$

But the second of these is also redundant: using the definition of the index of refreaction and Snell's law, we see that

$$\begin{aligned}\frac{n_1}{\mu_1} &= \frac{n_1^2}{\mu_1}\frac{1}{n_1} \\ &= \epsilon_1 \frac{\sin\theta_I}{n_2\sin\theta_T} \\ &= \frac{n_2\epsilon_1\sin\theta_I}{n_2^2\sin\theta_T} \\ &= \left(\frac{\epsilon_1\sin\theta_I}{\epsilon_2\sin\theta_T}\right)\frac{n_2}{\mu_2}\end{aligned}$$

so, in fact, this equation is that same as we got from the D_z condition.

So the only remaining unsolved equations are

$$\tilde{E}_{0R} - \tilde{E}_{0T} = 0, \qquad n_1\tilde{E}_{0R}\cos\theta_R/\mu_1 + n_2\tilde{E}_{0T}\cos\theta_T/\mu_2 = 0,$$

and since we assume all angles are strictly less that $\pi/2$ (the pathological case I mentioned earlier), the only solution to these is $\tilde{E}_{0R} = \tilde{E}_{0T} = 0$: like the incident electric field, the reflected and transmitted electric fields have no y-component and are thus given entirely in terms of the expressions for $\mathcal{E}_R$ and $\mathcal{E}_T$ above.

And we're done, at least for this particular case. In the next problem set, I'll ask you to do the case where the incident field has *only* a y-component, $\tilde{\vec{E}}_0 = \mathcal{E}_I\,\hat{e}_y$. It's slightly less tedious than this case, but still uses the same idea of matching the appropriate components of the various fields in medium 1 to those in medium 2.

4. Reflection and Transmission Coefficients

Now let's take a look at how the energy carried by the incident wave behaves. We know that the time-averaged power flowing through a surface element $\mathrm{d}\vec{\sigma}$ is $\mathrm{d}\bar{P} = \langle\vec{S}\rangle \cdot \mathrm{d}\vec{\sigma}$. If the surface element is on the boundary between two media, and the incident wave is $\vec{E}_I = \mathrm{Re}[\tilde{\vec{E}}_0 e^{i(\vec{k}_I\cdot\vec{r}-\omega t)}]$, then the time-averaged incident energy current is $\langle\vec{S}_I\rangle = \vec{k}_I|\tilde{\vec{E}}_0|^2/2\mu_1\omega$ and thus the rate at which the incident wave deposits energy onto this bit of boundary is

$$\begin{aligned} \mathrm{d}\bar{P}_I &= \langle \vec{S}_I \rangle \cdot \mathrm{d}\vec{\sigma} \\ &= \frac{\vec{k}_I |\tilde{\vec{E}}_0|^2}{2\mu_1 \omega} \cdot \mathrm{d}\vec{\sigma} \\ &= \frac{n_1}{2\mu_1} |\tilde{\vec{E}}_0|^2 \mathrm{d}\sigma \cos\,\theta_I \end{aligned}$$

where θ_I is the angle between the incident wave's direction and the normal to the surface.

Now we ask, what's the rate at which this energy *leaves* this bit of boundary? Since the reflected wave is also in medium 1 and has amplitude vector $\tilde{\vec{E}}_{0R}$, we have $\langle \vec{S}_R \rangle = \vec{k}_R |\tilde{\vec{E}}_{0R}|^2 / 2\mu_1 \omega$ and thus the reflected power is

$$\begin{aligned} \mathrm{d}\bar{P}_R &= \langle \vec{S}_R \rangle \cdot \mathrm{d}\vec{\sigma} \\ &= \frac{n_1}{2\mu_1} |\tilde{\vec{E}}_{0R}|^2 \mathrm{d}\sigma \cos\,\theta_R. \end{aligned}$$

The transmitted wave is in medium 2 and has amplitude vector $\tilde{\vec{E}}_{0T}$ so $\langle \vec{S}_T \rangle = \vec{k}_T |\tilde{\vec{E}}_{0R}|^2 / 2\mu_2 \omega$ and so the transmitted power is

$$\begin{aligned} \mathrm{d}\bar{P}_T &= \langle \vec{S}_T \rangle \cdot \mathrm{d}\vec{\sigma} \\ &= \frac{n_2}{2\mu_2} |\tilde{\vec{E}}_{0T}|^2 \mathrm{d}\sigma \cos\,\theta_T. \end{aligned}$$

We now define the reflection and transmission coefficients, denoted R and T respectively, to be the ratios of the reflected and transmitted powers to the incident power, i.e.

$$\begin{aligned} R &= \frac{\mathrm{d}\bar{P}_R}{\mathrm{d}\bar{P}_I} = \frac{|\tilde{\vec{E}}_{0R}|^2}{|\tilde{\vec{E}}_0|^2}, \\ T &= \frac{\mathrm{d}\bar{P}_T}{\mathrm{d}\bar{P}_I} = \frac{n_2 \mu_1 \cos\theta_T}{n_1 \mu_2 \cos\theta_I} \frac{|\tilde{\vec{E}}_{0T}|^2}{|\tilde{\vec{E}}_0|^2} \end{aligned}$$

(where we've used $\theta_I = \theta_R$).

In the absence of free charges/currents, all of the energy hitting the boundary must be accounted for by the reflected and transmitted energy, so conservation of energy requites $\mathrm{d}\bar{P}_I = \mathrm{d}\bar{P}_R + \mathrm{d}\bar{P}_T$, namely, $R + T = 1$. (If there are free charges, then some of the energy could be carried away in the form of kinetic energy and so $R + T < 1$.) Let's compute these coefficients for the example we considered in the previous section and make sure this identity holds.

With $\tilde{\vec{E}}_0 = \mathcal{E}_I(\cos\theta_I\,\hat{e}_x - \sin\theta_I\,\hat{e}_z)$, we found $\tilde{\vec{E}}_{0R} = -\mathcal{E}_R(\cos\theta_R\,\hat{e}_x + \sin\theta_R\,\hat{e}_z)$ and $\tilde{\vec{E}}_{0T} = \mathcal{E}_T(\cos\theta_T\,\hat{e}_x - \sin\theta_T\,\hat{e}_z)$ with

$$
\begin{aligned}
\mathcal{E}_R &= \left(\frac{n_2\epsilon_2\cos\theta_I - n_1\epsilon_1\cos\theta_T}{n_2\epsilon_2\cos\theta_I + n_1\epsilon_1\cos\theta_T}\right)\mathcal{E}_I, \\
\mathcal{E}_T &= \left(\frac{2n_1\epsilon_1\cos\theta_I}{n_2\epsilon_2\cos\theta_I + n_1\epsilon_1\cos\theta_T}\right)\mathcal{E}_I.
\end{aligned}
$$

The reflection coefficient is therefore

$$
\begin{aligned}
R &= \frac{|\tilde{\vec{E}}_{0R}|^2}{|\tilde{\vec{E}}_0|^2}, \\
&= \left(\frac{n_1\epsilon_2\cos\theta_I - n_2\epsilon_1\cos\theta_T}{n_1\epsilon_2\cos\theta_I + n_2\epsilon_1\cos\theta_T}\right)^2
\end{aligned}
$$

and the transmission coefficient is

$$
\begin{aligned}
T &= \frac{n_2\mu_1\cos\theta_T}{n_1\mu_2\cos\theta_I}\,\frac{|\tilde{\vec{E}}_{0T}|^2}{|\tilde{\vec{E}}_0|^2} \\
&= \frac{n_2\mu_1\cos\theta_T}{n_1\mu_2\cos\theta_I}\,\frac{4n_2^2\epsilon_1^2\cos^2\theta_I}{(n_1\epsilon_2\cos\theta_I + n_2\epsilon_1\cos\theta_T)^2} \\
&= \frac{n_2^2\mu_1\epsilon_1}{n_1^2\mu_2\epsilon_2}\,\frac{4n_1n_2\epsilon_1\epsilon_2\cos\theta_I\cos\theta_T}{(n_1\epsilon_2\cos\theta_I + n_2\epsilon_1\cos\theta_T)^2} \\
&= \frac{4n_1n_2\epsilon_1\epsilon_2\cos\theta_I\cos\theta_T}{(n_1\epsilon_2\cos\theta_I + n_2\epsilon_1\cos\theta_T)^2}
\end{aligned}
$$

since $n_1^2/n_2^2 = \mu_1\epsilon_1/\mu_2\epsilon_2$. You should be able to quickly show that

$$
(n_1\epsilon_2\cos\theta_I - n_2\epsilon_1\cos\theta_T)^2 + 4n_1n_2\epsilon_1\epsilon_2\cos\theta_I\cos\theta_T = (n_1\epsilon_2\cos\theta_I + n_2\epsilon_1\cos\theta_T)^2
$$

and this will allow you to confirm that $R + T = 1$ for this particular case (as it must be in all cases with no free charges or currents).