

$\underline{b}_1, \underline{b}_2$: primitive

$$\underline{b}'_1 = p \underline{b}_1 + q \underline{b}_2 \quad p, q, r, s$$

$$\underline{b}'_2 = r \underline{b}_1 + s \underline{b}_2 \quad \text{all integers}$$

$$\text{If } ps - qr = \pm 1$$

then \underline{b}'_1 and \underline{b}'_2 are primitive

$$\sqrt{b} = |\underline{b}_1 \times \underline{b}_2|$$

$$|\underline{b}'_1 \times \underline{b}'_2| = |(p \underline{b}_1 + q \underline{b}_2)$$

$$\times (r \underline{b}_1 + s \underline{b}_2)|$$

$$= |ps(\underline{b}_1 \times \underline{b}_2) + (qr \underline{b}_2 \times \underline{b}_1)|$$

$$= |ps - qr| |\underline{b}_1 \times \underline{b}_2| = (ps - qr) \sqrt{b}$$

$$b_1 = \frac{4\pi}{\sqrt{3}a} \left(\frac{\sqrt{3}}{2} \hat{x} + \hat{y} \right)$$

$$b_2 = \frac{4\pi}{\sqrt{3}a} \hat{y}$$

$$b_1' = b_1 - b_2 = \frac{4\pi}{\sqrt{3}a} \left(\frac{\sqrt{3}}{2} \hat{x} - \hat{y} \right)$$

$$b_2' = b_2$$

$$p=1, q=-1, r=1, s=0$$

$$|pq-rs| = 1$$



