EP402/MP464 – Solid State Physics

1st Assignment

Due in before 5pm on Friday 5th March 2021

1. Prove that the reciprocal lattice of a reciprocal lattice is the original lattice. In other words, if $(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$ are the basis vectors for a Bravais lattice and $(\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3)$ the corresponding basis for its recipocal lattice, then define vectors $(\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3)$ as

$$\begin{aligned} \mathbf{c}_1 &:= \frac{2\pi}{V_b}\,\mathbf{b}_2 \times \mathbf{b}_3, \\ \mathbf{c}_2 &:= \frac{2\pi}{V_b}\,\mathbf{b}_3 \times \mathbf{b}_1, \\ \mathbf{c}_3 &:= \frac{2\pi}{V_b}\,\mathbf{b}_1 \times \mathbf{b}_2. \end{aligned}$$

   Show that $\mathbf{c}_1 = \mathbf{a}_1$, $\mathbf{c}_2 = \mathbf{a}_2$ and $\mathbf{c}_3 = \mathbf{a}_3$.

   Note: It is this property of the Bravais lattice that allows us to use the reciprocal lattice vectors obtained via x-ray crystallography to determine the actual Bravais lattice of the solid under examination.

2. A 2-dimensional hexagonal crystal (for example graphene) is defined by Bravais lattice basis vectors

$$\mathbf{a}_1 = a\hat{\mathbf{x}}, \qquad \mathbf{a}_2 = -\frac{a}{2}\hat{\mathbf{x}} + \frac{\sqrt{3}\,a}{2}\hat{\mathbf{y}}.$$

   (a) Find basis vectors $\mathbf{b}_1$ and $\mathbf{b}_2$ for the reciprocal lattice, and use these to draw the the reciprocal lattice. (To do this, let $\mathbf{a}_3 = \hat{\mathbf{z}}$ be a "dummy" third Bravais lattice basis vector, and then use the usual formulae to get $\mathbf{b}_1$ and $\mathbf{b}_2$.)

   (b) On a separate diagram construct and draw the first and second Brillouin zones of this lattice.