Solutions to EE112 Autumn Repeat Exam '16-'17

P.1

(a) Here we just apply the various vector operations we've learned to the three vectors given...

(i) $\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$

$= (0)(1) + (-4)(0) + (-2)(-1) = \boxed{2}$ [3 marks]

(ii) $\vec{c}\times\vec{b} = (2\hat{i}+\hat{j}+\hat{k})\times(\hat{i}-\hat{k})$

$= 2(\hat{i}\times\hat{i}) + (\hat{j}\times\hat{i}) + (\hat{k}\times\hat{i}) - 2(\hat{i}\times\hat{k}) - (\hat{j}\times\hat{k}) - (\hat{k}\times\hat{k})$

$= (2)(\vec{0}) + (-\hat{k}) + (\hat{j}) - 2(-\hat{j}) - (\hat{i}) + (\vec{0})$

$= \boxed{-\hat{i}+3\hat{j}-\hat{k}}$ [3 marks]

(iii) $\hat{b}\times\hat{k} = (\hat{i}-\hat{k})\times\hat{k} = -\hat{j}$, so

$\hat{j}\times(\hat{b}\times\hat{k}) = \hat{j}\times(-\hat{j}) = \vec{0}$ and thus

$[\hat{j}\times(\hat{b}\times\hat{k})]\cdot\vec{a} = \boxed{0}$ [3 marks]

(iv) $\vec{c}\cdot\vec{a} = (2\hat{i}+\hat{j}+\hat{k})\cdot(-4\hat{j}-2\hat{k}) = -4-2 = -6$,

and so

$2(\vec{c}\cdot\vec{a})\hat{j} + 3\vec{a} = 2(-6)\hat{j} + 3(-4\hat{j}-2\hat{k})$

$= -12\hat{j} - 12\hat{j} - 6\hat{k}$

$= \boxed{-24\hat{j}-6\hat{k}}$ [3 marks]

(b) To do this, we just put the coordinate functions $x(t)$, $y(t)$ and $z(t)$ given by $\vec{r}(t)$ into the eq'n of the plane, then solve for t, then put this value back into $\vec{r}(t)$, like so:

$x(t) - 2y(t) + z(t) = (1+2t) - 2(2t) + (-t)$

$= 1-3t$

which must be 7 if the point is on the plane as well as the line. Thus, $1-3t = 7 \Rightarrow t = -2$. Therefore, the point where the line and plane intersect is

$\vec{r}(-2) = (1+2(-2))\hat{i} + 2(-2)\hat{j} - (-2)\hat{k}$

$= \boxed{-3\hat{i}-4\hat{j}+2\hat{k}}$ [3 marks]

(c) (i) We'll use the shifting theorem for this, which states that the LT of $e^{at}f(t)$ is $F(s-a)$ (where $F(s)$ is the LT of $f(t)$).

Here, we see

$L[e^{-2t}(2t^2+t)] = 2L[e^{-2t}t^2] + L[e^{-2t}t]$

and since the LT of t^2 is $\frac{2}{s^3}$ and of t is $\frac{1}{s^2}$, then
$L[e^{-2t}t^2] = \frac{2}{(s+2)^3}$ and $L[e^{-2t}t] = \frac{1}{(s+2)^2}$, so
$$L[e^{-2t}(2t^2+t)] = \boxed{\frac{4}{(s+2)^3} + \frac{1}{(s+2)^2}}$$ [4 marks]

(ii) $L^{-1}\left[\frac{3s-2}{s^2+9}\right] = 3L^{-1}\left[\frac{s}{s^2+9}\right] - 2L\left[\frac{1}{s^2+9}\right]$, so if we had the inverse LTs of $\frac{s}{s^2+9}$ and $\frac{1}{s^2+9}$, we'd be done. Looking at the table given, we see that $\frac{s}{s^2+\omega^2}$ is the LT of $\cos(\omega t)$, and thus $L^{-1}\left[\frac{s}{s^2+9}\right] = \cos(3t)$. Also, $\frac{\omega}{s^2+\omega^2}$ is the LT of $\sin(\omega t)$, and so $\frac{1}{s^2+\omega^2}$ is the LT of $\frac{1}{\omega}\sin\omega t$. Therefore, $\frac{1}{s^2+9^2}$ is the LT of $\frac{1}{3}\sin(3t)$, so $L^{-1}\left[\frac{1}{s^2+9}\right] = \frac{1}{3}\sin(3t)$. And so we're done:
$$L^{-1}\left[\frac{3s-2}{s^2+9}\right] = \boxed{3\cos(3t) - \frac{2}{3}\sin(3t)}$$ [4 marks]

(d) This just tests simple matrix operations:

(i)
$$BA = \begin{pmatrix} 5 & 0 \\ 0 & -2 \\ -8 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} (5)(3)+(0)(-1) & (5)(2)+(0)(1) \\ (0)(3)+(-2)(-1) & (0)(2)+(-2)(1) \\ (-8)(3)+(3)(-1) & (-8)(2)+(3)(1) \end{pmatrix}$$
$$= \boxed{\begin{pmatrix} 15 & 10 \\ 2 & -2 \\ -27 & -13 \end{pmatrix}}$$ [2 marks]

(ii) A^T is just A with the rows and columns switched:
$$A^T = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}^T = \begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 3 & -1 \\ 2 & 1 \end{pmatrix}}$$ [2 marks]

(iii) Ditto for B^T:
$$B^T = \begin{pmatrix} 5 & 0 \\ 0 & -2 \\ -8 & 3 \end{pmatrix}^T = \begin{pmatrix} 5 & 0 & -8 \\ 0 & -2 & 3 \end{pmatrix} = \boxed{\begin{pmatrix} 5 & 0 & -8 \\ 0 & -2 & 3 \end{pmatrix}}$$ [2 marks]

(iv) We have B^T from (iii), so let's use it:
$$A(B^T) = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 5 & 0 & -8 \\ 0 & -2 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} (3)(5)+(2)(0) & (3)(0)+(2)(-2) & (3)(-8)+(2)(3) \\ (-1)(5)+(1)(0) & (-1)(0)+(1)(-2) & (-1)(-8)+(1)(3) \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 15 & -4 & -18 \\ -5 & -2 & 11 \end{pmatrix}}$$ [2 marks]

(e) The trace is easy: just sum up the diagonal entries...

$$\mathrm{tr}\begin{pmatrix} 5 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 7 & -3 \end{pmatrix} = 5 + 1 + (-3) = \boxed{3}$$ [2 marks]

The determinant is best found by looking for a row or column with zeroes in it; let's pick the and use that:

$$\det\begin{pmatrix} 5 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 7 & -3 \end{pmatrix} = \begin{vmatrix} 5 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 7 & -3 \end{vmatrix} = -(0)\begin{vmatrix} 0 & 1 \\ 7 & -3 \end{vmatrix} + (1)\begin{vmatrix} 5 & 1 \\ 2 & -3 \end{vmatrix} - (0)\begin{vmatrix} 5 & 0 \\ 2 & 7 \end{vmatrix}$$

$$= (5)(-3) - (1)(2)$$ [4 marks]

$$= \boxed{-17}$$

(f) First we put the two eq'ns into matrix form:

$$x + 2y = 2, \; -x + y = 0 \implies \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}.$$

This has the form $A \cdot X = B$, with $A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \end{pmatrix}$, $B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$.
We want X, so if A^{-1} exists, we can use $X = A^{-1}B$.
$\det A = \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = (1)(1) - (2)(-1) = 3 \neq 0$, so A^{-1} exists and so we can find X uniquely. Recall that for a 2×2 matrix,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

so $A^{-1} = \frac{1}{3}\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$. Thus,

$$X = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} (1)(2) + (-2)(0) \\ (1)(2) + (1)(0) \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

so $\boxed{x = y = 2/3}$ is the sol'n. [6 marks]

(g) First we need the characteristic polynomial $p_M(\lambda)$ of the matrix M.
This is defined as $p_M(\lambda) = \det(M - \lambda I)$, or

$$p_M(\lambda) = \det\left(\begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right) = \begin{vmatrix} 2-\lambda & -3 \\ 1 & -2-\lambda \end{vmatrix}$$

$= (2-\lambda)(-2-\lambda) - (-3)(1) = -4 + \lambda^2 + 3 = \lambda^2 - 1.$

The Cayley-Hamilton Thm says that the characteristic eqn of a matrix M is $p_M(M) = 0$, so here we have $\boxed{M^2 - I = 0}$ as this matrix's characteristic eq'n. [4 marks]

To use this to find M^{-1}, note that $p_M(0) = -1$, and this is the determinant of M, so M^{-1} exists. Thus, multiplying the characteristic eq'n by M^{-1} gives, $M^{-1}(M^2 - I) = M - M^{-1} = 0$,

or

$$\boxed{M^{-1} = M = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}}$$ [3 marks]

(In other words, this particular matrix is its own inverse!)

P.2

(a) First, we LT the entire eq'n: we have $y(t) \to Y(s) = L[y(t)]$ and $\frac{dy}{dt}(t) \to sY(s) - y(0) = sY(s)$ (since $y(0) = 0$ here).

Using the table in the back, $\sinh(2t) \to \frac{2}{s^2-4}$, so the eq'n becomes

$$\frac{dy}{dt} - 2y = 3\sinh(2t) \longrightarrow sY(s) - 2Y(s) = \frac{6}{s^2-4}$$

which gives the transfer function

$$Y(s) = \frac{6}{(s-2)(s^2-4)} = \frac{6}{(s+2)(s-2)^2}.$$

So if we can find a function that has $\frac{6}{(s+2)(s-2)^2}$ as its LT, we have $y(t)$.

There are several ways we can do this, but let's use the partial fraction expansion method: we know there must exist three constants A, B and C such that

$$\frac{6}{(s+2)(s-2)^2} = \frac{A}{s+2} + \frac{B}{s-2} + \frac{C}{(s-2)^2}$$

Multiplying both sides by $(s+2)(s-2)^2$ gives

$$\begin{aligned} 6 &= A(s-2)^2 + B(s+2)(s-2) + C(s+2) \\ &= A(s^2 - 4s + 4) + B(s^2 - 4) + C(s+2) \\ &= (A+B)s^2 + (-4A + C)s + (4A - 4B + 2C) \end{aligned}$$

Now, both sides must hold for all values of s, which means the coefficients of s^n on both sides must match: There is no s^2 term on the LHS, so $A + B = 0$; There's no s term either, so $-4A + C = 0$. And the constant term on the LHS is 6 and on the RHS is $4A - 4B + 2C$, so

$4A-4B+2C=6$. Thus, $B=-A$ and $C=4A$, so putting these into the third eq'n gives $4A-4(-A)+2(4A)=16A=6$, so $A=3/8$. Thus, $B=-3/8$ and $C=3/2$, so

$$\frac{6}{(s+2)(s-2)^2}=\frac{3/8}{s+2}-\frac{3/8}{s-2}+\frac{3/2}{(s-2)^2}.$$

The first is the LT of $\frac{3}{8}e^{-2t}$ and the second of $-\frac{3}{8}e^{2t}$; we know that $\frac{1}{s^2}$ is the LT of t, so using the shift theorem, $\frac{1}{(s-2)^2}$ is the LT of te^{2t}, so $L^{-1}\left[\frac{3/2}{(s-2)^2}\right]=\frac{3}{2}te^{2t}$. Putting everything together, we have our sol'n:

$$y(t)=LT^{-1}[Y(s)]=L^{-1}\left[\frac{3/8}{s+2}-\frac{3/8}{s-2}+\frac{3/2}{(s-2)^2}\right]$$

$$=\boxed{\frac{3}{8}\left(e^{-2t}-e^{2t}\right)+\frac{3}{2}te^{2t}=\frac{3}{2}te^{2t}-\frac{3}{8}\sinh(2t)}$$

[10 marks]

(b) First, we need the characteristic . . . . . :

$$\det A=\begin{vmatrix}4-\lambda & 0 & 6\\ 0 & -3-\lambda & 0\\ 4 & 0 & 2-\lambda\end{vmatrix}=(4-\lambda)\begin{vmatrix}-3-\lambda & 0\\ 0 & 2-\lambda\end{vmatrix}-0\begin{vmatrix}0 & 0\\ 4 & 2-\lambda\end{vmatrix}+6\begin{vmatrix}0 & -3-\lambda\\ 4 & 0\end{vmatrix}$$

$$=(4-\lambda)(-3-\lambda)(2-\lambda)+24(-3-\lambda)$$
$$=(-3-\lambda)\left[8-6\lambda+\lambda^2-24\right]=-(\lambda+3)(\lambda^2-6\lambda-16)$$
$$=-(\lambda+3)(\lambda+2)(\lambda-8)$$

and so we can immediately see the three values of λ for which this vanishes – i.e. the eigenvalues – are $\boxed{\lambda_1=-3,\ \lambda_2=-2 \text{ and } \lambda_3=8.}$ [2 marks each]

Now, each eigenvector K_i satisfies $AK_i=\lambda_i K_i$ and has the form $K_i=\begin{pmatrix}a_i\\ b_i\\ c_i\end{pmatrix}$, so we can find a_i, b_i and c_i (up to an overall multiplicative constant) by looking for solns to $(A-\lambda_i I)K_i=0$.

Let's do this:

$\underline{\lambda_1=-3}$: $A-\lambda_1 I=\begin{pmatrix}4-(-3) & 0 & 6\\ 0 & -3-(-3) & 0\\ 4 & 0 & 2-(-3)\end{pmatrix}=\begin{pmatrix}7 & 0 & 6\\ 0 & 0 & 0\\ 4 & 0 & 5\end{pmatrix}$

so if $K_1=\begin{pmatrix}a_1\\ b_1\\ c_1\end{pmatrix}$, then

$$\begin{pmatrix}7 & 0 & 6\\ 0 & 0 & 0\\ 4 & 0 & 5\end{pmatrix}\begin{pmatrix}a_1\\ b_1\\ c_1\end{pmatrix}=\begin{pmatrix}7a_1+6c_1\\ 0\\ 4a_1+5c_1\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$$

so $7a_1+6c_1=0$ and $4a_1+5c_1=0$. The first gives $c_1=-\frac{7}{6}a_1$, and putting this into the second gives $4a_1+5(-7/6\,a_1) = -\frac{11}{6}a_1 = 0$, so $a_1=0$. Thus, $c_1=0$ as well. Thus, K_1 has the form $\begin{pmatrix}0\\b_1\\0\end{pmatrix}$ for any constant b_1. We can choose b_1 to be whatever we like (except 0) because all eigenvectors are only determined up to an overall multiplicative factor, so let's choose $b_1=1$, to get $\boxed{K_1=\begin{pmatrix}0\\1\\0\end{pmatrix}}$ as the eigenvector [3 marks] associated with $\lambda_1=-3$

$\underline{\lambda_2=-2}$: $A-\lambda_2 I = \begin{pmatrix}6&0&6\\0&-1&0\\4&0&4\end{pmatrix}$, so if $K_2=\begin{pmatrix}a_2\\b_2\\c_2\end{pmatrix}$ then

$\begin{pmatrix}6&0&6\\0&-1&0\\4&0&4\end{pmatrix}\begin{pmatrix}a_2\\b_2\\c_2\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \Rightarrow 6a_2+6c_2=0,\ -b_2=0,\ 4a_2+4c_2=0$. The second gives $b_2=0$, and both the first and the third give $c_2=-a_2$, so K_2 has the form $\begin{pmatrix}a_2\\0\\-a_2\end{pmatrix}$. Again, any vector of this form is an eigenvector, so pick $a_2=1$ to get $\boxed{K_2=\begin{pmatrix}1\\0\\-1\end{pmatrix}}$. [3 marks]

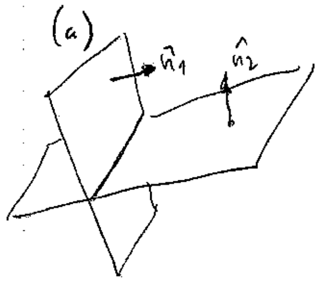
$\underline{\lambda_3=8}$: $A-\lambda_3 I = \begin{pmatrix}-4&0&6\\0&-11&0\\4&0&-6\end{pmatrix}$, and taking $K_3=\begin{pmatrix}a_3\\b_3\\c_3\end{pmatrix}$

gives $\begin{pmatrix}-4&0&6\\0&-11&0\\4&0&-6\end{pmatrix}\begin{pmatrix}a_3\\b_3\\c_3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix} \Rightarrow -4a_3+6c_3=0,\ -11b_3=0,\ 4a_3-6c_3=0$.
The second gives $b_3=0$ and both the first and third give $c_3=\frac{2}{3}a_3$, so $K_3=\begin{pmatrix}a_3\\0\\2/3\,a_3\end{pmatrix}$. Any choice of a_3 will do, so if $a_3=3$, we get

$\boxed{K_3=\begin{pmatrix}3\\0\\2\end{pmatrix}}$ as an eigenvector associated to $\lambda_3=8$. [3 marks]

P.3

(a)



Two intersecting planes are shown to the left, and we want the line which is formed by their intersection. This line must be perpendicular to both normal vectors $\hat{n}_1$ and $\hat{n}_2$, so its direction vector is $\vec{a}=\hat{n}_1\times\hat{n}_2$.

Recall that all planes have the form $\hat{n}\cdot\vec{r} = \text{const}$, so simply looking at the equation of each plane gives its normal vector: thus, the normal vector to the plane $x+y+z=1$ is $\hat{n}_1=\hat{i}+\hat{j}+\hat{k}$, and $x-2z=0$ has normal vector $\hat{n}_2=\hat{i}-2\hat{k}$. Thus,

$\vec{a} = \hat{n}_1 \times \hat{n}_2 = (\hat{i}+\hat{j}+\hat{k}) \times (\hat{i} - 2\hat{k}) = -2\hat{i} + 3\hat{j} - \hat{k}$.

Now, if we can find any point $\vec{r_0}$ that lies in both planes, we have that $\vec{r}(t) = \vec{a}t + \vec{r_0}$ gives the line of intersection. If a point lies in the second plane, then $x = 2z$. This means that, if it also lies in the first plane, $x+y+z = (2z)+y+z = y+3z = 1$, so any choice of y & z which satisfy this will do. So, for example, $y=1$, $z=0$. Since $x = 2z$, $x = 0$ as well, so the point $\vec{r}_0 = \hat{j}$ lies in both planes. Therefore the line which is formed by the planes' intersection is $\vec{r}(t) = \vec{a}t + \vec{r_0}$ $= (-2\hat{i}+3\hat{j}-\hat{k})t + (\hat{j})$, or, in parametric form,

$$\boxed{x(t) = -2t, \quad y(t) = 3t+1, \quad z(t) = -t}$$ [10 marks]

where t ranges over all real numbers.

(b) Let's use Gauss–Jordan reduction to find the inverse of this matrix: first, we form the 3×6 matrix obtained by appending the identity matrix to the given matrix:

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 2 & 4 & 9 & 0 & 0 & 1 \end{array}\right)$$

The trick is to now use row operations so the LHS 3×3 matrix reduces to $\left(\begin{smallmatrix}1&0&0\\0&1&0\\0&0&1\end{smallmatrix}\right)$. Whatever is in the RHS is then the inverse of the original matrix. So, let's add -2 times the 1st row to the 3rd row:

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 0 & 0 & -1 & -2 & 0 & 1 \end{array}\right)$$

Multiply the 2nd and 3rd rows by -1: $\left(\begin{array}{ccc|ccc} 1 & 2 & 5 & 1 & 0 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right)$

Add -2 times the 2nd row to the 1st: $\left(\begin{array}{ccc|ccc} 1 & 0 & 9 & 1 & 2 & 0 \\ 0 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right)$

Add -9 times the 3rd row to the 1st: $\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -17 & 2 & 9 \\ 0 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right)$

Add 2 times the 3rd row to the 2nd: $\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -17 & 2 & 9 \\ 0 & 1 & 0 & 4 & -1 & -2 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right)$.

The left-hand 3×3 part is now the identity matrix, as desired. So the 3×3 part on the right is the inverse we're after, so

$$\boxed{\begin{pmatrix} 1 & 2 & 5 \\ 0 & -1 & 2 \\ 2 & 4 & 9 \end{pmatrix}^{-1} = \begin{pmatrix} -17 & 2 & 9 \\ 4 & -1 & -2 \\ 2 & 0 & -1 \end{pmatrix}}$$ [15 marks]

P.4

(a) Use Gauss-Jordan form! First, we write the eq'ns in matrix form:

$2I_1+I_2=-4$, $I_1-I_2+I_3=0$, $2I_1-2I_3=8$

$$\Downarrow$$

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & -2 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 8 \end{pmatrix}.$$

So the augmented matrix is obtained by appending the vector on the RHS to the coefficient matrix, i.e. $\left(\begin{array}{ccc|c} 2 & 1 & 0 & -4 \\ 1 & -1 & 1 & 0 \\ 2 & 0 & -2 & 8 \end{array}\right)$. We now want to use row operations to reduce the left 3×3 part to upper triangular form, then look at the simplified eq'ns and solve them. So let's add -1 times the 1st row to the 3rd:

$$\left(\begin{array}{ccc|c} 2 & 1 & 0 & -4 \\ 1 & -1 & 1 & 0 \\ 0 & -1 & -2 & 12 \end{array}\right)$$

Now, $-1/2$ times the 1st to the 2nd:
$$\left(\begin{array}{ccc|c} 2 & 1 & 0 & -4 \\ 0 & -3/2 & 1 & 2 \\ 0 & -1 & -2 & 12 \end{array}\right)$$

Now, $-2/3$ times the 2nd to the 3rd:
$$\left(\begin{array}{ccc|c} 2 & 1 & 0 & -4 \\ 0 & -3/2 & 1 & 2 \\ 0 & 0 & -8/3 & 32/3 \end{array}\right)$$

The resulting eq'ns are given by $\begin{pmatrix} 2 & 1 & 0 \\ 0 & -3/2 & 1 \\ 0 & 0 & -8/3 \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 32/3 \end{pmatrix}$, or

$2I_1+I_2=-4$, $-\frac{3}{2}I_2+I_3=2$, $-\frac{8}{3}I_3=32/3$. The last immediately gives $\boxed{I_3=-4}$. The second says that $-\frac{3}{2}I_2=2-I_3$, [4 marks]

or

$$\boxed{I_2=-\frac{2}{3}(2-I_3)=-4}$$ [4 marks]

and the first says

$$\boxed{I_1=\frac{1}{2}(-4-I_2)=0}$$ [4 marks]

and so we have our currents.

(b) As stated in the table, $s = \int_{t_1}^{t_2} |\vec{u}(t)|\, dt$, where $\vec{u}(t) = \frac{d\vec{r}}{dt}(t)$ is the tangent vector of the curve at point t. For this particular curve,

$$\vec{u}(t) = \frac{d}{dt}\left[\sin^3(t)\,\hat{i} - \cos^3(t)\,\hat{k}\right]$$

$$= 3\sin^2(t)\cos(t)\,\hat{i} + 3\cos^2(t)\sin(t)\,\hat{k}$$

and so

$$|\vec{u}(t)| = \sqrt{u_x^2 + u_y^2 + u_z^2} = \sqrt{9\sin^4(t)\cos^2(t) + 9\cos^4(t)\sin^2(t)}$$

$$= \sqrt{9\sin^2(t)\cos^2(t)\left[\sin^2(t) + \cos^2(t)\right]}$$

$$= \sqrt{9\sin^2(t)\cos^2(t)}$$

$$= 3|\sin(t)\cos(t)|$$

Now, since $0 \le t \le \pi/2$, both $\sin(t)$ and $\cos(t)$ are positive, so $|\sin(t)\cos(t)| = \sin(t)\cos(t)$. Therefore, the arc length of this curve is

$$s = \int_0^{\pi/2} 3\sin(t)\cos(t)\,dt = \int_0^{\pi/2} 3\sin(t)\, d(\sin(t))$$

$$= \frac{3}{2}\sin^2(t)\Big|_0^{\pi/2} = \frac{3}{2}\left[\sin^2(\tfrac{\pi}{2}) - \sin^2(0)\right]$$

$$= \boxed{3/2}$$

[13 marks]