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THE NATIONAL UNIVERSITY OF IRELAND MAYNOOTH

BE in Electronic Engineering with Communications
BE in Electronic Engineering with Computers
BE in Electronic Engineering
BE in Robotics and Intelligent Devices

YEAR 1

Autumn Repeat Exam

2016–2017

Engineering Mathematics II
EE112

Dr. P. Watts

Time allowed: 2 **hours**

Answer Question 1 and any two others

Question 1 carries 50 marks and all others carry 25 marks each

1. **This Question Is Compulsory**

(a) [**12 marks**] If $\vec{a} = -4\hat{\jmath} - 2\hat{k}$, $\vec{b} = \hat{\imath} - \hat{k}$ and $\vec{c} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$, obtain the following:

$$\begin{array}{ll} \text{(i)} & \vec{a} \cdot \vec{b}, \\ \text{(ii)} & \vec{c} \times \vec{b}, \\ \text{(iii)} & [\hat{\jmath} \times (\vec{b} \times \hat{k})] \cdot \vec{a}, \\ \text{(iv)} & 2(\vec{c} \cdot \vec{a})\hat{\jmath} + 3\vec{a}. \end{array}$$

(b) [**3 marks**] Find the point at which the line $\vec{r}(t) = (1 + 2t)\hat{\imath} + 2t\hat{\jmath} - t\hat{k}$ (where t is a real number) and the plane $x - 2y + z = 7$ intersect.

(c) [**8 marks**] Find the following Laplace transform and inverse Laplace transform:

$$\begin{array}{ll} \text{(i)} & L\left[e^{-2t}\left(2t^2 + t\right)\right], \\ \text{(ii)} & L^{-1}\left[\dfrac{3s - 2}{s^2 + 9}\right]. \end{array}$$

(d) [**8 marks**] For the two matrices

$$A = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 5 & 0 \\ 0 & -2 \\ -8 & 3 \end{pmatrix},$$

Find (i) BA, (ii) A^{T}, (iii) B^{T} and (iv) $A(B^{\mathrm{T}})$.

(e) [**6 marks**] Find the determinant and trace of the matrix

$$\begin{pmatrix} 5 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 7 & -3 \end{pmatrix}.$$

(f) [**6 marks**] Solve the following system of simultaneous equations using the inverse matrix method:

$$x + 2y = 2, \qquad -x + y = 0.$$

(g) [**7 marks**] Find the characteristic equation for the matrix

$$M = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}.$$

and use it to compute M^{-1}.

2. (a) [**10 marks**] Solve the following differential equation using Laplace transforms:

$$\frac{\mathrm{d}y}{\mathrm{d}t} - 2y = 3\sinh(2t)$$

where $y(0) = 0$.

(b) [**15 marks**] Find the eigenvalues of the matrix

$$\begin{pmatrix} 4 & 0 & 6 \\ 0 & -3 & 0 \\ 4 & 0 & 2 \end{pmatrix}$$

and determine their associated eigenvectors.

3. (a) [**10 marks**] Find the line of intersection, expressed in parametric form, between the planes $x + y + z = 1$ and $x - 2z = 0$.

(b) [**15 marks**] Using any method you like, find the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 5 \\ 0 & -1 & 2 \\ 2 & 4 & 9 \end{pmatrix}.$$

4. (a) [**12 marks**] A particular circuit has three resistors such that the currents I_1, I_2 and I_3 passing through them satisfy the equations

$$\begin{aligned} 2I_1 + I_2 &= -4, \\ I_1 - I_2 + I_3 &= 0, \\ 2I_1 - 2I_3 &= 8. \end{aligned}$$

Find I_1, I_2 and I_3 using Gauss-Jordan elimination.

(b) [**13 marks**] Consider the curve is given by

$$\vec{r}(t) = \sin^3(t)\hat{\imath} - \cos^3(t)\hat{k},$$

for $0 \leq t \leq \pi/2$. Find the total arc length of this curve.

USEFUL FORMULAE

Vectors

$$\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \left(\vec{A} \cdot \vec{C}\right)\vec{B} - \left(\vec{A} \cdot \vec{B}\right)\vec{C}$$

curvature: $\kappa = \dfrac{\left|\frac{\mathrm{d}\hat{u}}{\mathrm{d}t}\right|}{|\vec{u}|}$

principal unit normal vector: $\hat{N} = \dfrac{\frac{\mathrm{d}\hat{u}}{\mathrm{d}t}}{\left|\frac{\mathrm{d}\hat{u}}{\mathrm{d}t}\right|}$

arc length between t_1 and t_2: $s = \displaystyle\int_{t_1}^{t_2} |\vec{u}(t)|\,\mathrm{d}t$

Laplace Transforms

Table of Laplace Transforms

$f(t) = L^{-1}[F(s)]$	$F(s) = L[f(t)]$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$

Laplace Transform Theorems

$$\begin{aligned}
L\left[af(t) + bg(t)\right] &= aL\left[f(t)\right] + bL\left[g(t)\right], \\
L\left[e^{at}f(t)\right] &= F(s-a), \\
L\left[f(at)\right] &= \frac{1}{a}F\left(\frac{s}{a}\right), \\
L\left[f'(t)\right] &= sF(s) - f(0), \\
L\left[f''(t)\right] &= s^2F(s) - sf(0) - f'(0), \\
L\left[\int_0^t f(\tau)\,\mathrm{d}\tau\right] &= \frac{1}{s}F(s).
\end{aligned}$$

In all of the above, $n = 0, 1, 2, \ldots$ and ω, a and b are constants.