EE112 – Engineering Mathematics II

Problem Set 8

Due by 5pm on Friday, 13 April 2018

1. Comsider the following matrices:

$$A = \begin{pmatrix} 7 & -1 \end{pmatrix}, \ B = \begin{pmatrix} 8 & -6 \\ 1 & 4 \end{pmatrix}, \qquad C = \begin{pmatrix} 8 & 0 & -4 \\ 1 & 1 & 1 \end{pmatrix}, \ D = \begin{pmatrix} 1 \\ -3 \end{pmatrix}.$$

   State if each of the following mathematical expressions makes sense according to the rules of matrix algebra. For those that do, compute them.
   (i) $2A - B$,
   (ii) BC,
   (iii) $DA - 2B$;
   (iv) C^2.

2. If x and y are numbers, we know that $(x + y)^2 = x^2 + 2xy + y^2$. However, this formula does *not* hold for matrices due to the fact that matrix multiplication is not commutative. Here we derive the correct formula.

   (a) If A and B are two square matrices, show that $(A + B)^2 = A^2 + AB + BA + B^2$.

   (b) Confirm the above formula works for the matrices

$$A = \begin{pmatrix} 0 & -1 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -4 & -1 \\ 2 & 2 & -2 \end{pmatrix}$$

   in the following way:
   (i) First, compute the sum $A+B$ and then square it to get $(A+B)^2$.
   (ii) Now, compute A^2, AB, BA and B^2 separately.
   (iii) Finally, compute $A^2 + AB + BA + B^2$ and confirm it's the same as (i).

3. Now take the two matrices in previous problem and compute both $A^2 + 2AB + B^2$ and $A^2 + 2BA + B^2$. Show that not only are they different from each other, but neither one is the matrix $(A + B)^2$ you computed in Problem 2. This illustrates the importance of remembering that matrix multiplication is *not* commutative.