EE112 – Engineering Mathematics II

Problem Set 10

Due by 5pm on Friday, 27 April 2018

1. (a) Compute the determinant of the following two 4×4 matrices:

   (i) $\begin{pmatrix} 5 & 0 & 0 & 0 \\ 2 & 8 & 0 & 0 \\ 3 & 0 & 6 & 0 \\ 1 & -1 & 2 & \frac{1}{10} \end{pmatrix}$ (ii) $\begin{pmatrix} 0 & -1 & 0 & 4 \\ -3 & 0 & 0 & 1 \\ 0 & 3 & -5 & 0 \\ -1 & 0 & 2 & 0 \end{pmatrix}$

   (b) Suppose A and B are two $m \times m$ matrices whose product AB is the $m \times m$ zero matrix. Explain why this means that at least one of $\det(A)$ and $\det(B)$ must be zero.

2. Show that the following pairs of matrices are inverses of each other.

   (a) $\begin{pmatrix} -3 & 6 \\ 5 & -9 \end{pmatrix}$, $\begin{pmatrix} 3 & 2 \\ \frac{5}{3} & 1 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{pmatrix}$, $\frac{1}{2}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{pmatrix}$

3. (a) State whether or not the following matrices are invertible and justify your answer:

   (i) $\begin{pmatrix} 5 & -1 & 0 \\ -3 & 7 & 7 \end{pmatrix}$,   (ii) $\begin{pmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ 2 & 0 & 3 \end{pmatrix}$,   (iii) $\begin{pmatrix} 2 & i \\ 3i & 1 \end{pmatrix}$

   (b) Show that the two matrices below are rotation matrices:

   (i) $\frac{1}{7}\begin{pmatrix} 2 & 3 & 6 \\ 3 & -6 & 2 \\ 6 & 2 & -3 \end{pmatrix}$ (ii) $\begin{pmatrix} -\sqrt{3}/2 & -1/2 \\ 1/2 & -\sqrt{3}/2 \end{pmatrix}$

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